Supplemental Appendix

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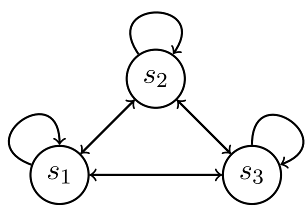
1 Model description

Let $\boldsymbol{y}^{(i)}$ be an $n_i \times 1$ vector corresponding to the respiratory sinus arrhythmia (RSA) measurements for a participant $i \in \{1, 2, \dots, N\}$ where n_i represents the number of observations for participant i. Let $\boldsymbol{b}^{(i)}$ represent the corresponding latent RSA state sequence for participant i where for each time point $k \in \{1, 2, \dots, n_i\}$, $b_k^{(i)} \in \{1, 2, 3\}$ meaning the latent RSA state at any given instance in time can be from one of three states where $b_k^{(i)} = 1$ corresponds to *baseline*. For the purposes of fitting the hidden Markov model (HMM), we will assume that the true, latent RSA state of each participant is *baseline* during the pre-stressor/baseline phase of the experiment (i.e. $b_j^{(i)} = 1$ for $j = 1, 2, \dots, k^\star - 1$ where $k^\star$ is the time in which the stressor is introduced). As mentioned in the manuscript, the interpretation of states 2 and 3 need not correspond to *reactivity* and *recovery* because we allow all transitions between the three states. The possible state transitions can be illustrated with the directed graph (left) and adjacency matrix (right) below:



$$\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \end{array}$$

where

$$s_1 = \text{state 1 } (\textit{baseline}), \qquad s_2 = \text{state 2}, \qquad s_3 = \text{state 3}.$$

Lastly, there exists three covariates of interest that we believe can effect the RSA measurements: (1) age, (2) sex, and (3) indicator of if the participant's parent has a Bachelor's degree or higher (edu). Lastly, to address the second aim of the manuscript, Daily Life Experiences of Racism (DLER) is included in the HMM as a predictor (details below) to quantify the effect of prior exposure to racial discrimination on parasympathetic nervous system (PNS) response. Note that age and DLER are centered when they are used in the HMM.

1.1 Markov model for latent RSA state sequence

The repeated RSA measurements are processed such that the observations occur on a 30 second grid. Therefore, we can treat time as discrete since the inter-observation times are equal for all participants. Within the HMM, we model $\boldsymbol{b}^{(i)}$ as a discrete time, time homogeneous Markov process defined by a transition probability matrix, $\mathbf{P}$, given by

$$\mathbf{P} = \begin{pmatrix} \frac{1}{1+e^{q_1}+e^{q_2}} & 0 & 0 \\ 0 & \frac{1}{1+e^{q_3}+e^{q_4}} & 0 \\ 0 & 0 & \frac{1}{1+e^{q_5}+e^{q_6}} \end{pmatrix} \cdot \begin{pmatrix} 1 & e^{q_1} & e^{q_2} \\ e^{q_3} & 1 & e^{q_4} \\ e^{q_5} & e^{q_6} & 1 \end{pmatrix}.$$

In order to address all research aims, we will consider three different parameterizations of q_j, for $j \in \{1, \dots, 6\}$:

$$\begin{array}{ll} \textit{Aim 1} & q_j = \zeta_{0,j} + \zeta_{1,j} \cdot \text{age}_i + \zeta_{2,j} \cdot \mathbb{1}(\text{sex}_i = 1) + \zeta_{3,j} \cdot \mathbb{1}(\text{edu}_i = 1) \\ \textit{Aim 2 (covariates)} & q_j = \zeta_{0,j} + \zeta_{1,j} \cdot \text{age}_i + \zeta_{2,j} \cdot \mathbb{1}(\text{sex}_i = 1) + \zeta_{3,j} \cdot \mathbb{1}(\text{edu}_i = 1) + \zeta_{4,j} \cdot \text{DLER}_i \\ \textit{Aim 2 (only DLER)} & q_j = \zeta_{0,j} + \zeta_{1,j} \cdot \text{DLER}_i \end{array}$$

Let $\boldsymbol{\zeta}_j$ represent the vector of $\zeta_{\cdot,j}$ coefficients for each aim and for $j \in \{1, 2, \dots, 6\}$.

1.2 Random effect mean response model

The RSA response of a participant i at a given time k, $y_k^{(i)}$, is modeled as a Gaussian random variable with a Gaussian random effect mean, conditional on the RSA state, $b_k^{(i)}$ (i.e. state-specific parent means and parent standard deviations are defined for each RSA state). Hence, for each of the respective research aims, we have

$$\begin{array}{ll} \textit{Aim 1} & y_k^{(i)} = \boldsymbol{x}^{(i)}\boldsymbol{\gamma} + u_1^{(i)} \cdot \mathbb{1}(b_k^{(i)} = 1) + u_2^{(i)} \cdot \mathbb{1}(b_k^{(i)} = 2) + u_3^{(i)} \cdot \mathbb{1}(b_k^{(i)} = 3) + \epsilon_k^{(i)} \\ \textit{Aim 2 (covariates)} & y_k^{(i)} = \boldsymbol{x}^{(i)}\boldsymbol{\gamma} + (u_1^{(i)} + \text{DLER}_i \cdot \delta_1) \cdot \mathbb{1}(b_k^{(i)} = 1) + \\ & \quad (u_2^{(i)} + \text{DLER}_i \cdot \delta_2) \cdot \mathbb{1}(b_k^{(i)} = 2) + (u_3^{(i)} + \text{DLER}_i \cdot \delta_3) \cdot \mathbb{1}(b_k^{(i)} = 3) + \epsilon_k^{(i)} \\ \textit{Aim 2 (only DLER)} & y_k^{(i)} = (u_1^{(i)} + \text{DLER}_i \cdot \delta_1) \cdot \mathbb{1}(b_k^{(i)} = 1) + (u_2^{(i)} + \text{DLER}_i \cdot \delta_2) \cdot \mathbb{1}(b_k^{(i)} = 2) + \\ & \quad (u_3^{(i)} + \text{DLER}_i \cdot \delta_3) \cdot \mathbb{1}(b_k^{(i)} = 3) + \epsilon_k^{(i)} \end{array}$$

where $\epsilon_k^{(i)} \overset{iid}{\sim} N(0, \tau^2)$ for all i, k, $u_1^{(i)} \overset{iid}{\sim} N(\mu, \sigma_1^2)$, $u_2^{(i)} \overset{iid}{\sim} N(\mu+\alpha, \sigma_2^2)$, $u_3^{(i)} \overset{iid}{\sim} N(\mu+\beta, \sigma_3^2)$, and

$$\boldsymbol{x}^{(i)} = \begin{pmatrix} \text{age}_i & \mathbb{1}(\text{sex}_i = 1) & \mathbb{1}(\text{edu}_i = 1) \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}^T.$$

Now, let $\boldsymbol{b}_s^{(i)}$ be a $n_i \times 1$ vector defined as $b_k^{(i)} = s$, $\forall k \in \{1, \dots, n_i\}$, and $s \in \{1, 2, 3\}$. Then, the distribution of $\boldsymbol{y}^{(i)}$ takes the following form according to the different aims:

$$\begin{aligned} &\textit{Aim 1}: \\ &\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_1^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \sigma_1 \sim \mathrm{N}_{n_i}\Big[(\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mu) \cdot \mathbf{1}_{n_i}, \quad \sigma_1^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\Big] \\ &\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_2^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \sigma_2 \sim \mathrm{N}_{n_i}\Big[(\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mu + \alpha) \cdot \mathbf{1}_{n_i}, \ \sigma_2^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\Big] \\ &\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_3^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \sigma_3 \sim \mathrm{N}_{n_i}\Big[(\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mu + \beta) \cdot \mathbf{1}_{n_i}, \ \sigma_3^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\Big] \end{aligned} \tag{1}$$

Aim 2 (covariates) :

$$\begin{aligned}
&\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_1^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_1, \sigma_1 \sim \mathrm{N}_{n_i}\Big[(\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mathrm{DLER}_i \cdot \delta_1 + \mu) \cdot \mathbf{1}_{n_i}, \qquad \sigma_1^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\Big]\\
&\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_2^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_2, \sigma_2 \sim \mathrm{N}_{n_i}\Big[(\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mathrm{DLER}_i \cdot \delta_2 + \mu + \alpha) \cdot \mathbf{1}_{n_i}, \; \sigma_2^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\Big]\\
&\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_3^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_3, \sigma_3 \sim \mathrm{N}_{n_i}\Big[(\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mathrm{DLER}_i \cdot \delta_3 + \mu + \beta) \cdot \mathbf{1}_{n_i}, \; \sigma_3^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\Big]
\end{aligned} \tag{2}$$

Aim 2 (only DLER) :

$$\begin{aligned}
&\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_1^{(i)}, \mu, \alpha, \beta, \tau, \delta_1, \sigma_1 \sim \mathrm{N}_{n_i}\big[(\mathrm{DLER}_i \cdot \delta_1 + \mu) \cdot \mathbf{1}_{n_i}, \qquad \sigma_1^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\big]\\
&\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_2^{(i)}, \mu, \alpha, \beta, \tau, \delta_2, \sigma_2 \sim \mathrm{N}_{n_i}\big[(\mathrm{DLER}_i \cdot \delta_2 + \mu + \alpha) \cdot \mathbf{1}_{n_i}, \; \sigma_2^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\big]\\
&\boldsymbol{y}^{(i)} \mid \boldsymbol{b}_3^{(i)}, \mu, \alpha, \beta, \tau, \delta_3, \sigma_3 \sim \mathrm{N}_{n_i}\big[(\mathrm{DLER}_i \cdot \delta_3 + \mu + \beta) \cdot \mathbf{1}_{n_i}, \; \sigma_3^2 \cdot \mathbf{J}_{n_i} + \tau^2 \cdot \mathbf{I}_{n_i}\big]
\end{aligned}$$

where $\mathbf{1}_{n_i}$ is a column vector of ones with length n_i, $\mathbf{J}_{n_i} = \mathbf{1}_{n_i} \cdot \mathbf{1}_{n_i}^T$, and $\mathbf{I}_{n_i}$ is the identity matrix of dimension $n_i \times n_i$. Recall that a canonical modeling assumption for HMMs is assuming the response observations over time are conditionally independent of one another given the latent state sequence (i.e., $(y_k^{(i)} \mid b_k^{(i)}) \perp (y_j^{(i)} \mid b_j^{(i)})$ for $k \neq j$ and $k, j \in \{1, 2, \ldots, n_i\}$). However, due to the random effects structure in modeling the mean of the RSA response, this conditional independence assumption no longer holds for all $k \in \{1, 2, \ldots, n_i\}$. Hence, define

$$\boldsymbol{y}_1^{(i)} := \{y_k^{(i)}\}_{k\,:\,b_k^{(i)}=1}, \quad \boldsymbol{y}_2^{(i)} := \{y_k^{(i)}\}_{k\,:\,b_k^{(i)}=2}, \quad \boldsymbol{y}_3^{(i)} := \{y_k^{(i)}\}_{k\,:\,b_k^{(i)}=3}.$$

In other words, $\boldsymbol{y}_s^{(i)}$ for $s \in \{1, 2, 3\}$ is the vector of RSA measurements corresponding to the indices in which the latent RSA state sequence, $\boldsymbol{b}^{(i)}$, takes the value s. Consequently, let

$$n_{i,1} := \sum_{k=1}^{n_i} \mathbb{1}(b_k^{(i)} = 1), \quad n_{i,2} := \sum_{k=1}^{n_i} \mathbb{1}(b_k^{(i)} = 2), \quad n_{i,3} := \sum_{k=1}^{n_i} \mathbb{1}(b_k^{(i)} = 3).$$

Then, in conjunction with (1) and (2), we can write

Aim 1:

$$\begin{aligned}
&\boldsymbol{y}_1^{(i)} \mid \{b_k^{(i)} = 1\}_{k=1}^{n_1}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \sigma_1 \sim \mathrm{N}_{n_1}\Big((\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mu) \cdot \mathbf{1}_{n_1}, \qquad \sigma_1^2 \cdot \mathbf{J}_{n_1} + \tau^2 \cdot \mathbf{I}_{n_1}\Big)\\
&\boldsymbol{y}_2^{(i)} \mid \{b_k^{(i)} = 2\}_{k=1}^{n_2}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \sigma_2 \sim \mathrm{N}_{n_2}\Big((\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mu + \alpha) \cdot \mathbf{1}_{n_2}, \; \sigma_2^2 \cdot \mathbf{J}_{n_2} + \tau^2 \cdot \mathbf{I}_{n_2}\Big)\\
&\boldsymbol{y}_3^{(i)} \mid \{b_k^{(i)} = 3\}_{k=1}^{n_3}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \sigma_3 \sim \mathrm{N}_{n_3}\Big((\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mu + \beta) \cdot \mathbf{1}_{n_3}, \; \sigma_3^2 \cdot \mathbf{J}_{n_3} + \tau^2 \cdot \mathbf{I}_{n_3}\Big)
\end{aligned}$$

Aim 2 (covariates):

$$\begin{aligned}
&\boldsymbol{y}_1^{(i)} \mid \{b_k^{(i)} = 1\}_{k=1}^{n_1}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_1, \sigma_1 \sim \mathrm{N}_{n_1}\Big((\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mathrm{DLER}_i \cdot \delta_1 + \mu) \cdot \mathbf{1}_{n_1}, \qquad \sigma_1^2 \cdot \mathbf{J}_{n_1} + \tau^2 \cdot \mathbf{I}_{n_1}\Big)\\
&\boldsymbol{y}_2^{(i)} \mid \{b_k^{(i)} = 2\}_{k=1}^{n_2}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_2, \sigma_2 \sim \mathrm{N}_{n_2}\Big((\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mathrm{DLER}_i \cdot \delta_2 + \mu + \alpha) \cdot \mathbf{1}_{n_2}, \; \sigma_2^2 \cdot \mathbf{J}_{n_2} + \tau^2 \cdot \mathbf{I}_{n_2}\Big)\\
&\boldsymbol{y}_3^{(i)} \mid \{b_k^{(i)} = 3\}_{k=1}^{n_3}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_3, \sigma_3 \sim \mathrm{N}_{n_3}\Big((\boldsymbol{x}^{(i)}\boldsymbol{\gamma} + \mathrm{DLER}_i \cdot \delta_3 + \mu + \beta) \cdot \mathbf{1}_{n_3}, \; \sigma_3^2 \cdot \mathbf{J}_{n_3} + \tau^2 \cdot \mathbf{I}_{n_3}\Big)
\end{aligned}$$

Aim 2 (only DLER):

$$\boldsymbol{y}_1^{(i)} \mid \{b_k^{(i)} = 1\}_{k=1}^{n_1}, \mu, \alpha, \beta, \tau, \delta_1, \sigma_1 \sim \mathrm{N}_{n_1}\big((\mathrm{DLER}_i \cdot \delta_1 + \mu) \cdot \mathbf{1}_{n_1}, \quad \sigma_1^2 \cdot \mathbf{J}_{n_1} + \tau^2 \cdot \mathbf{I}_{n_1}\big)$$
$$\boldsymbol{y}_2^{(i)} \mid \{b_k^{(i)} = 2\}_{k=1}^{n_2}, \mu, \alpha, \beta, \tau, \delta_2, \sigma_2 \sim \mathrm{N}_{n_2}\big((\mathrm{DLER}_i \cdot \delta_2 + \mu + \alpha) \cdot \mathbf{1}_{n_2}, \ \sigma_2^2 \cdot \mathbf{J}_{n_2} + \tau^2 \cdot \mathbf{I}_{n_2}\big)$$
$$\boldsymbol{y}_3^{(i)} \mid \{b_k^{(i)} = 3\}_{k=1}^{n_3}, \mu, \alpha, \beta, \tau, \delta_3, \sigma_3 \sim \mathrm{N}_{n_3}\big((\mathrm{DLER}_i \cdot \delta_3 + \mu + \beta) \cdot \mathbf{1}_{n_3}, \ \sigma_3^2 \cdot \mathbf{J}_{n_3} + \tau^2 \cdot \mathbf{I}_{n_3}\big)$$

1.3 Likelihood expression

Using the information from Section 1.1 and 1.2, we can write the likelihood for participant i as follows:

$$\begin{aligned} &f(\boldsymbol{y}^{(i)} \mid \boldsymbol{b}^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_1, \delta_2, \delta_3, \sigma_1, \sigma_2, \sigma_3, \{\boldsymbol{\zeta}_j\}_{j=1}^{6}) \\ &\quad = \left(\prod_{s=1}^{3} f(\boldsymbol{y}_s^{(i)} \mid \{b_k^{(i)} = s\}_{k=1}^{n_s}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_s, \sigma_s)\right)\left(\pi^T_{b_1^{(i)}} \cdot \prod_{k=2}^{n_i} \mathbf{P}_{b_{k-1}^{(i)}, b_k^{(i)}}\right) \end{aligned}$$

where π is a 3×1 initial state probability vector. Since we assume that a participant's true RSA state sequence before the introduction of a stressor is *baseline* (state 1), we know $\pi^T = (1, 0, 0)$. Then, the full joint likelihood expression is given by

$$f(\boldsymbol{y}^{(1)}, \boldsymbol{y}^{(2)}, \ldots, \boldsymbol{y}^{(N)}) = \prod_{i=1}^{N} f(\boldsymbol{y}^{(i)} \mid \boldsymbol{b}^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_1, \delta_2, \delta_3, \sigma_1, \sigma_2, \sigma_3, \{\boldsymbol{\zeta}_j\}_{j=1}^{6}).$$

Parameter estimation is done through a Metropolis-Hastings update via Bayesian Markov chain Monte Carlo (MCMC) sampling techniques. Assuming the prior distribution for each parameter is an uninformed Gaussian distribution, we can write the joint posterior as

$$\begin{aligned} &\pi\Big(\boldsymbol{b}^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_1, \delta_2, \delta_3, \sigma_1, \sigma_2, \sigma_3, \{\boldsymbol{\zeta}_j\}_{j=1}^{6} \mid \{\boldsymbol{y}^{(i)}\}_{i=1}^{N}\Big) \\ &\quad \propto \left[\prod_{i=1}^{N} f(\boldsymbol{y}^{(i)} \mid \boldsymbol{b}^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_1, \delta_2, \delta_3, \sigma_1, \sigma_2, \sigma_3, \{\boldsymbol{\zeta}_j\}_{j=1}^{6}\right] \\ &\qquad \times \pi(\boldsymbol{b}^{(i)}, \mu, \alpha, \beta, \boldsymbol{\gamma}, \tau, \delta_1, \delta_2, \delta_3, \sigma_1, \sigma_2, \sigma_3, \{\boldsymbol{\zeta}_j\}_{j=1}^{6}) \end{aligned}$$

Note that $\boldsymbol{b}^{(i)}$ is treated as a sequence of unknown parameters, thus using rejection sampling, we can estimate its posterior distribution. Additionally, recall that we assume $b_k^{(i)} = 1$ for all $k < k^\star$ where $k^\star$ is the time in which the stressor is introduced. Thus, we only need to sample $b_k^{(i)}$ for $k \in \{k^\star, k^\star + 1, \ldots, n_i\}$.