Homework #9

[Your name]

[Total - 100 pts]

Question 1: Recall from Example 9.2 the following table:

x	0	1	2	3	4	5
$f(x;1/2)$	1/32	5/32	10/32	10/32	5/32	1/32
$f(x;3/4)$	1/1024	15/1024	90/1024	270/1024	405/1024	243/1024
$\frac{f(x;1/2)}{f(x;3/4)}$	32/1	32/3	32/9	32/27	32/81	32/243

Find the best critical region, C, of size $\alpha = \frac{6}{32}$.

[1] Solution ■

Question 2: Let $X_1, \ldots, X_n \overset{iid}{\sim} f(x)$ with support $\{0, 1, \ldots\}$. We wish to test the following hypotheses:

$$H_0 \;:\; f(x) = \frac{e^{-1}}{x!} \cdot \mathbf{1}\{x \in \{0, 1, \ldots\}\}$$
$$H_1 \;:\; f(x) = \left(\frac{1}{2}\right)^{x+1} \cdot \mathbf{1}\{x \in \{0, 1, \ldots\}\}$$

That is, we want to test whether our data come from a Poisson distribution with mean $\lambda = 1$ versus a geometric distribution with $p = 1/2$.

(a) Find a best critical region C. Leave C in terms of $k > 0$ (from Remark 9.4).

(b) Consider the case of $k = 1$ and $n = 1$. Using R/Python/Software, determine the level of significance for the critical region from part (a). In other words, what is $P_{H_0}(\boldsymbol{X} \in C)$? [*Hint: it may be easier to compute* $1 - P_{H_0}(\boldsymbol{X} \in C^c)$ *because* $|C^c| = 2$]

[2.a] Solution ■

[2.b] Solution ■

Question 3: Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\theta, 1)$ with θ unknown. Determine if a uniformly most powerful test of the simple hypothesis $H_0 : \theta = \theta_0$ versus the composite hypothesis $H_1 : \theta \neq \theta_0$ exists. If it exists, define the rejection region C left in terms of $k > 0$. If it does *not* exist, explain why.

[3] Solution ■