Homework #7

[Your name]

[Total - 100 pts]

Question 1: Suppose $X_1, \ldots, X_n \sim \text{Unif}[0, \theta]$ for $\theta > 0$.

(a) Find a sufficient statistic Y for θ by using the factorization theorem.

(b) Given $\hat{\theta}^{\text{MME}} = 2\bar{X}$, calculate $\text{MSE}(\hat{\theta}^{\text{MME}}, \theta)$.

[1.a] Solution ■

[1.b] Solution ■

Question 2: Suppose $X_1, \ldots, X_n \sim \text{Pois}(\theta)$ for $\theta > 0$. Define $Y = X_1 + \cdots + X_n$, which we know is a sufficient statistic for $\theta > 0$. [*Hint: we know $Y \sim Poisson(n\theta)$*]

(a) Show Y is a complete sufficient statistic.

(b) Derive the UMVUE of θ.

[2.a] Solution ■

[2.b] Solution ■

Question 3: Given random samples $X_1, \ldots, X_n \sim \text{Geo}(p)$ with $0 < p < 1$ and $n \geq 2$. Note the probability mass function is given by

$$f(x; p) = (1-p)^{x-1} p, \quad x = 1, 2, \ldots, \quad 0 < p < 1.$$

(a) Find the complete sufficient statistic related to $p \in (0, 1)$

(b) Show that $\sum_{i=1}^{n} X_i$ follows negative binomial distribution, i.e.

$$P\left(\sum_{i=1}^{n} X_i = y\right) = \binom{y-1}{n-1} p^n (1-p)^{y-n}.$$

[*Hint: first consider the special case $n = 2$ and follow steps similar to Example 1.3 from Lecture 1; then extend to any n*]

(c) Find the UMVUE for p. [*Hint: $\mathbb{E}_p(\mathbf{1}\{X_1 = 1\}) = P_p(X_1 = 1) = p$ and Rao-Blackwell*]

[3.a] Solution ■

[3.b] Solution ■

[3.c] Solution ■

Question 4: Suppose $X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\theta, \sigma^2)$, $\theta \in (-\infty, \infty)$ and $\sigma^2 > 0$. Suppose σ^2 is known (i.e., we are not concerned with estimating it). Show that $\bar{X}$ is an efficient estimator for θ.

[4] Solution ■