Homework #5

[Your name]

`[Total - 100 pts]`

Question 1: Suppose $X_1, \dots, X_n \overset{iid}{\sim} \text{Bernoulli}(p)$ with $0 \le p \le 1$. Find the maximum likelihood estimator of p by following these steps.

(a) Find the log likelihood function for $0 < p < 1$. Note that if $p = 0, 1$, the log likelihood is not defined.

(b) Calculate the first and the second derivatives of the log likelihood function for $0 < p < 1$.

(c) Use Theorem 5.1 to conclude that $\hat{p} = \sum_{i=1}^{n} x_i/n$ is the maximizer of $l(p)$ for $0 < p < 1$.

(d) We can find that the likelihood is defined at $p = 0$ or 1. Show that $\hat{p} = \sum_{i=1}^{n} x_i/n$ is also the maximizer of the likelihood for $0 \le p \le 1$.

[1.a] Solution ■

[1.b] Solution ■

[1.c] Solution ■

[1.d] Solution ■

Question 2: Suppose $X_1, \dots, X_n \overset{iid}{\sim} \text{Unif}[0, \theta]$, where $0 < \theta < \infty$. Find the maximum likelihood estimator (MLE) of θ.

[2] Solution ■

Question 3: Suppose $X_1, \dots, X_n \overset{iid}{\sim} \text{Unif}[\theta_1 - \theta_2, \theta_1 + \theta_2]$ with $-\infty < \theta_1 < \infty, \theta_2 > 0$. Using a random sample, find the maximum likelihood estimators of the parameters θ_1, θ_2.

[3] Solution ■

Question 4: Suppose $X_1, \ldots, X_n$ are iid from the following distributions in (a) and (b). Find the maximum likelihood estimator (MLE) for θ and get its limiting distribution.

(a) Beta distribution Beta$(\theta, 1)$, $\theta > 0$ where the pdf is $f_X(x;\theta) = \theta x^{\theta-1} \cdot \mathbf{1}\{x \in (0,1)\}$.

(b) Pareto distribution Pareto$(1, \theta)$, $\theta > 2$ where the pdf is $f_X(x;\theta) = \theta x^{-\theta-1} \cdot \mathbf{1}\{x \in (1,\infty)\}$.

[4.a] Solution ■

[4.b] Solution ■