Homework #4

[Your Name]

[Total - 100 pts]

Question 1: Suppose that $X_1, \ldots, X_n$ are random samples from the following distributions. Find the method of moments estimators for each case.

(a) Gamma$(\alpha, 2)$, $\alpha > 0$. What is $\hat{\alpha}^{\text{MME}}$?

(b) Gamma$(2, \beta)$, $\beta > 0$. What is $\hat{\beta}^{\text{MME}}$?

[1.a] Solution ■

[1.b] Solution ■

Question 2: Suppose $X_1, \ldots, X_n \overset{iid}{\sim} \text{Ber}(p)$ with $0 \leq p \leq 1$.

(a) Find $\hat{p}^{\text{MME}}$.

(b) Use CLT to find the asymptotic distribution of $\hat{p}^{\text{MME}}$.

[2.a] Solution ■

[2.b] Solution ■

Question 3: Suppose $X_1, \ldots, X_n \overset{iid}{\sim} \text{Gamma}(\alpha, \beta)$ with $\alpha > 0, \beta > 0$, whose probability density function is given by:

$$f(x; \theta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \cdot \mathbf{1}_{(x>0)}, \quad \theta = (\alpha, \beta)^T.$$

Find $\hat{\alpha}^{\text{MME}}$ and $\hat{\beta}^{\text{MME}}$.

[3] Solution ■

Question 4: Suppose $X_1, \ldots, X_n \overset{iid}{\sim} \text{Beta}(\alpha, 1)$ with $\alpha > 0$, whose probability density function is given by:

$$f(x; \alpha) = \alpha x^{\alpha - 1} \cdot \mathbf{1}_{(0,1)}(x).$$

(a) Find $\hat{\alpha}^{\text{MME}}$.

(b) Find the asymptotic distribution of $\hat{\alpha}^{\text{MME}}$.

[4.a] Solution ■

[4.b] Solution ■

Question 5: Suppose $X_1, \ldots, X_n \overset{iid}{\sim} \text{Poisson}(\lambda)$ with $\lambda > 0$. Recall that we may have two estimators: $\hat{\lambda}_1^{\text{MME}} = \bar{X}$ and $\hat{\lambda}_2^{\text{MME}} = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$

(a) Find the asymptotic distribution of $\hat{\lambda}_1^{\text{MME}}$.

(b) Find the asymptotic distribution of $\hat{\lambda}_2^{\text{MME}}$. (*Hint 1*: use the result and procedure of Example 3.7 for guidance) (*Hint 2*: if $X \sim \text{Poisson}(\lambda)$, then $\mathbb{E}[X(X-1)(X-2)\cdots(X-k+1)] = \lambda^k$)

(c) Can we compare the efficiency of two estimators by comparing the asymptotic variance? State which estimator is better with some reasonable justification.

[5.a] Solution ■

[5.b] Solution ■

[5.c] Solution ■